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Homework No. 11
Finite Elements, Winter 2018/19

Problem 11.1: Elastic plates

"Plates" are flat bodies with (constant) positive, but small thickness. In linear Kirchhoff plate theory the vertical displacement of a plate can be described as solution u of the following fourth order PDE

$$\Delta^2 u = f, \quad \text{in } \Omega, \tag{11.1}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded domain and Δ^2 is the **biharmonic operator** defined by

$$\Delta^2 u(x,y) = \Delta(\Delta u(x,y)) = \partial_x^4 u(x,y) + 2\partial_x^2 \partial_y^2 u(x,y) + \partial_y^4 u(x,y).$$

(a) For the boundary conditions

$$\begin{aligned} u &= 0 \quad \text{on } \partial\Omega \\ \partial_{\boldsymbol{n}} u &= 0 \quad \text{on } \partial\Omega\,, \end{aligned}$$

derive a symmetric weak formulation of the plate equation (11.1). Here $\partial_{\boldsymbol{n}} u$ is the normal derivative.

(b) Prove the existence and uniqueness of a solution for $\Omega = (0,1)^2$.

Hint: Think of a useful variant of the Poincaré inequality.